AP CALCULUS AB
PREREQUISITE SUMMER ASSIGNMENT
& SURVIVAL TOOLKIT

To: All students enrolled in AP Calculus AB
From: Mrs. Curtiss, AP Calculus Teacher

AP Calculus AB is a college level course covering material traditionally taught in the first semester of college calculus. Going into AP calculus AB, there are certain skills that have been taught to you over the previous years that you are expected to have mastered. If you do not have these skills, you will find that you will consistently get problems incorrect next year, even though you understand the calculus concepts. It is frustrating for students when they are tripped up by the algebra and not the calculus. This summer packet is intended for you to brush up and possibly relearn these topics.

Students need a strong foundation to be ready for the rigorous work required throughout this course. Completing the prerequisite summer assignment should prepare you for the material. **This packet will be collected on the first day of school. You will be tested on this material the first week of school.** Show all work neatly in the space provided.

Rather than give you a textbook to remind you of the techniques necessary to solve the problem, below I have given you several websites that have full instructions on the techniques. If and when you are unsure of how to attempt these problems, examine these websites. **Don't fake your way through these problems.** As stated, students are notoriously weak in them, even students who have achieved well prior to AP calculus. Use the websites. You need to get off to a good start so *spend quality time on the packet this summer*. .

A graphing calculator is required on the AP test and is required for this course. However, **do not rely on your calculator**. Use the calculator only on the problems when absolutely necessary. **Half of the AP exam next year is taken without any calculator!**

It is a mistake to decide to do this all now. Visit this packet often throughout the entire summer. You need these techniques to be fresh in your mind in the fall. Also, do not wait to do them at the very last minute. These take time and effort. The listed topics in the review are listed on the next page. You can certainly do **Google** searches for any of these topics. But I have given you several sites that cover pretty much everything.

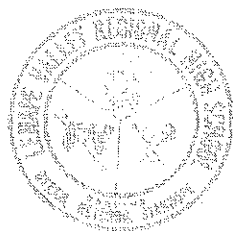Video lessons for every topic can be found and watched at:

http://mathispower4u.yolasite.com/

https://www.khanacademy.org/math/algebra2

Trig Information:
http://www.mathematicshelpcentral.com/index.html

Limits:
http://www.calculus-help.com/funstuff/phobe.html

Have a Great Summer!
Mrs. Curtiss

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SKILLS NEEDED FOR CALCULUS

I. Algebra:
- A. *Exponents (operations with integer, fractional, and negative exponents)
- B. *Factoring (GCF, trinomials, difference of squares and cubes, sum of cubes, grouping)
- C. Rationalizing (numerator and denominator)
- D. *Simplifying rational expressions
- E. *Solving algebraic equations and inequalities (linear, quadratic, higher order using synthetic division, rational, radical, and absolute value equations)
- F. Simultaneous equations (systems of equations)

II. Graphing and Functions:
- A. *Lines (intercepts, slopes, write equations using point-slope and slope-intercept, parallel, perpendicular, distance and midpoint formulas)
- B. Conic Sections (circle, parabola, ellipse, and hyperbola)
- C. *Functions (definition, notation, domain, range, inverse, composition)
- D. *Basic shapes and transformations of the following functions: absolute value, rational, root, higher order curves, *log*, *ln*, exponential, trigonometric, piece-wise, inverse functions)
- E. Tests for symmetry: even & odd.

III. Geometry:
- A. Pythagorean Theorem
- B. Area Formulas (circles, polygons, surface area of solids)
- C. Volume Formulas
- D. Similar Triangles

IV. *Logarithmic and Exponential Functions:
- A. *Simplify Expressions (Use laws of logarithms and exponents)
- B. *Solve exponential and logarithmic equations (include *ln* as well as *log*)
- C. *Sketch graphs
- D. *Inverses

V. *Trigonometry:
- A. *Unit Circle (definition of functions, angles in radians and degrees, *no calculator!)
- B. Use of Pythagorean Identities and formulas to simplify expressions and prove identities
- C. *Solve equations
- D. *Inverse Trigonometric Functions
- E. Right triangle trigonometry
- F. *Graphs

VI. *Functions and Models
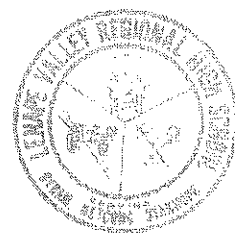- A. Using your Graphing Calculator.
- B. Problem Solving

VII. Limits:
- A. Concept of a limit
- B. Find limits as x approaches a number and as x approaches ∞

* A solid working foundation in these areas is extremely important!

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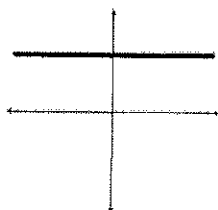# PREREQUISITE SUMMER ASSIGNMENT
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TOOLKIT OF FUNCTIONS
***STUDENTS SHOULD KNOW THE BASIC SHAPE OF THESE FUNCTIONS AND BE ABLE TO
GRAPH THEIR TRANSFORMATIONS WITHOUT THE ASSISTANCE OF A CALCULATOR!***
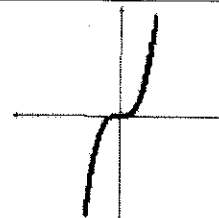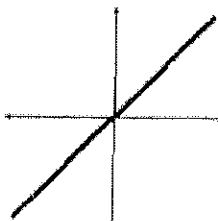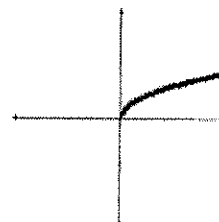
Constant

$$f(x) = a$$

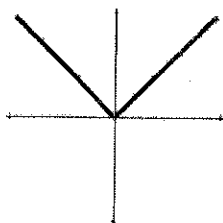Cubic

$$f(x) = x^3$$
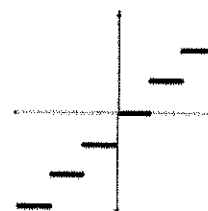
Identity

$$f(x) = x$$

Square Root

$$f(x) = \sqrt{x}$$

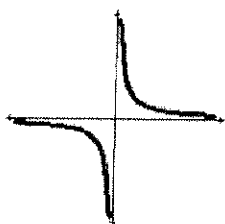Absolute Value

$$f(x) = |x|$$

Greatest Integer

$$f(x) = [x]$$
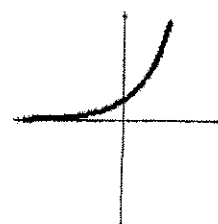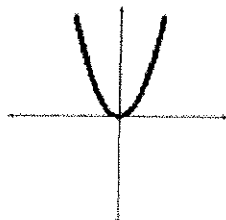
Reciprocal

$$f(x) = \frac{1}{x}$$
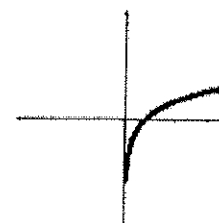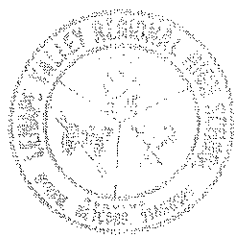
Exponential

$$f(x) = a^x$$

Quadratic

$$f(x) = x^2$$
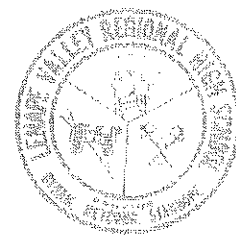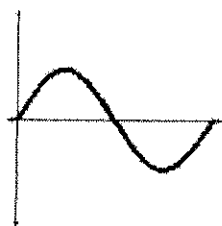
Logarithmic

$$f(x) = \ln x$$

Trig Functions
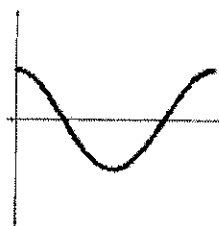
$f(x) = \sin x$ $f(x) = \cos x$ $f(x) = \tan x$
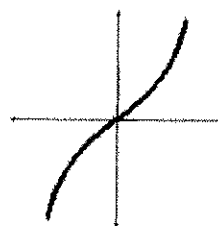
Polynomial Functions:

A function P is called a polynomial if $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$
Where n is a nonnegative integer and the numbers $a_0, a_1, a_2, \ldots a_n$ are constants.

Even degree	Odd degree
Leading coefficient sign	Leading coefficient sign
Positive Negative	Positive Negative
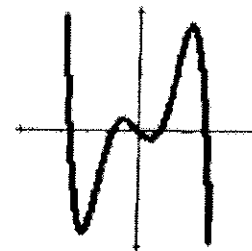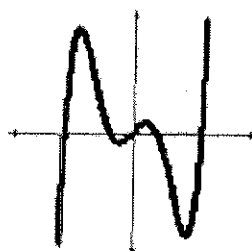
- Number of roots equals the degree of the polynomial.
- Number of x intercepts is less than or equal to the degree.
- Number of "bends" is less than or equal to (degree – 1).

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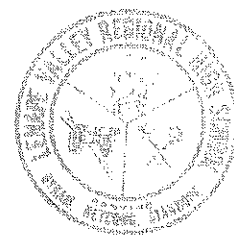
Trig Formulas:

Arc Length of a circle: $\qquad L = r\theta \quad$ or $\quad L = \dfrac{d}{360} \cdot 2\pi r$

Area of a sector of a circle: Area $= \dfrac{1}{2} r^2 \theta \quad$ or $\quad$ Area $= \dfrac{d}{360} \cdot \pi r^2$

Solving parts of a triangle:

Law of Sines: $\qquad\qquad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc \ \cos A$$
$$b^2 = a^2 + c^2 - 2ac \ \cos B$$
$$c^2 = a^2 + b^2 - 2ab \ \cos C$$

Area of a Triangle:

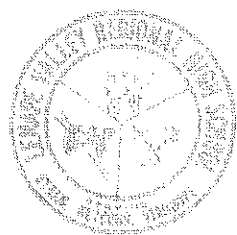$$\text{Area} = \dfrac{1}{2} bc \sin A \quad \text{or} \quad \text{Area} = \dfrac{1}{2} ac \sin B \quad \text{or} \quad \text{Area} = \dfrac{1}{2} ab \sin C$$

Hero's formula : Area $= \sqrt{s(s-a)(s-b)(s-c)}$, where s = semi perimeter

Ambiguous Case:

θ **is acute**

Compute: alt $=$ adj $\cdot \sin\theta$

opp $<$ alt   No triangle
opp $=$ alt   1 triangle (right)
opp$>$adj   1 triangle

alt$<$opp$<$adj   2 triangles

θ **is obtuse or right**

opp $\leq$ adj   No triangle
opp $>$ adj   1 triangle

Does a triangle exist?   Yes - when

(difference of 2 sides) $<$ (third side) $<$ (Sum of 2 sides)

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Trig Identities:

Reciprocal Identities:

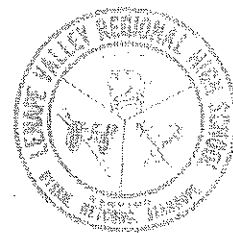$$\csc A = \frac{1}{\sin A} \qquad \sec A = \frac{1}{\cos A} \qquad \cot A = \frac{1}{\tan A}$$

Quotient Identities: $\qquad \tan A = \frac{\sin A}{\cos A} \qquad \cot A = \frac{\cos A}{\sin A}$

Pythagorean Identities:

$$\sin^2 A + \cos^2 A = 1 \quad \tan^2 A + 1 = \sec^2 A \qquad 1 + \cot^2 A = \csc^2 A$$

Sum and Difference Identities:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B \quad \sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B \quad \cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Double Angle Identities:

$$\sin(2A) = 2\sin A \cos A \qquad \tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$$

$$\cos(2A) = \cos^2 A - \sin^2 A \qquad \cos(2A) = 2\cos^2 A - 1 \quad \cos(2A) = 1 - 2\sin^2 A$$

Half Angle Identities:

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1 - \cos A}{2}} \qquad \cos\frac{A}{2} = \pm\sqrt{\frac{1 + \cos A}{2}} \qquad \tan\frac{A}{2} = \pm\sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

Polar Formulas:

$$x^2 + y^2 = r^2 \qquad x = r\cos\theta \qquad y = r\sin\theta \qquad \tan^{-1}\frac{y}{x} = \theta \quad x > 0, \quad \tan^{-1}\frac{y}{x} = 0 + \pi \quad x < 0$$

Geometric Formulas:

Area of a trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$ Area of a triangle: $A = \frac{1}{2}bh$

Area of an equilateral triangle: $A = \frac{\sqrt{3}}{4}s^2$

Area if a circle: $A = \pi r^2$ Circumference of a circle: $C = 2\pi r$ or $C = d\pi$
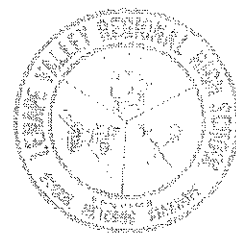
Unit Circle – Degrees and Radians
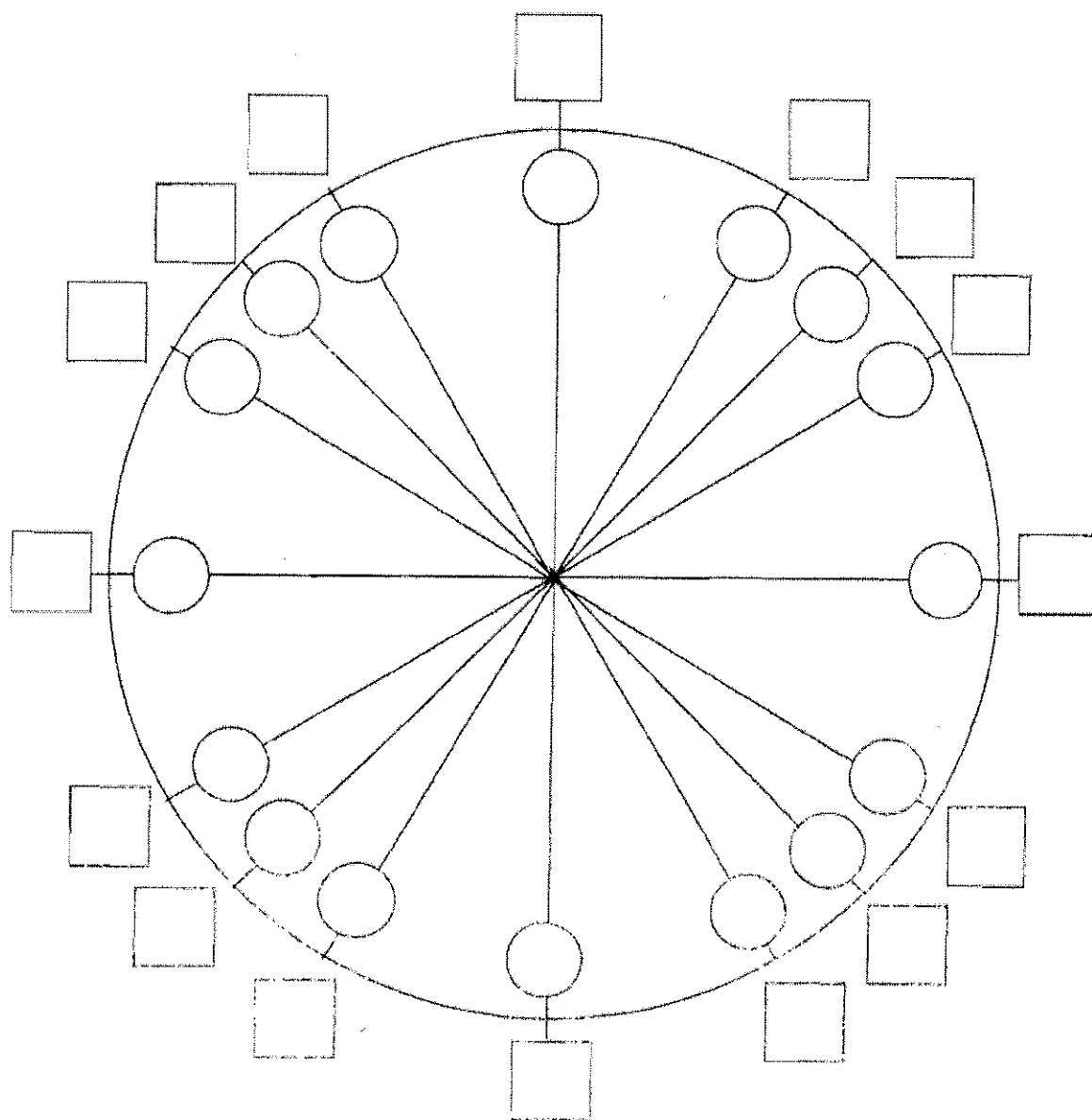
Place degree measures in the circles.

Place radian measure in the squares.

Place $(\cos\theta, \sin\theta)$ in parenthesis outside the square.

$\tan\theta =$ _____

$\cot\theta =$ _____

$\csc\theta =$ _____

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Prerequisite problems for Calculus:

Complete the following problems on your own paper. Show all work in a clear and neat manner. Clearly indicate your final answer. This assignment must be completed and turned in on the first day school.

I. Algebra:
A. Exponents
1. Simplify. Show all steps clearly.

a. $\dfrac{(8x^3yz)^{\frac{1}{3}}(2x)^3}{4x^{\frac{1}{3}}\left(yz^{\frac{2}{3}}\right)^{-1}}$

b. $h \div \dfrac{x+h}{h}$

c. $\dfrac{\sqrt{x-2}+\dfrac{5}{\sqrt{x-2}}}{x-2}$

d. $\dfrac{x^3-4x^2+2x+3}{x-3}$

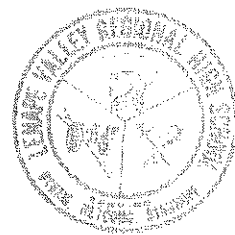e. $\dfrac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

f. $\left(\dfrac{1}{x^{-2}} + \dfrac{4}{x^{-1}y^{-1}} + \dfrac{1}{y^{-2}}\right)^{-\frac{1}{2}}$

B. Factor Completely
2.Factor
a. $9x^2 + 3x - 3xy - y$ (use grouping)

b. $64x^6 - 1$ (use diff. of squares first)

c. $15x^{\frac{5}{2}} - 2x^{\frac{3}{2}} - 24x^{\frac{1}{2}}$ (factor GCF $x^{\frac{1}{2}}$ first)

d. $x^{-1} - 3x^{-2} + 2x^{-3}$ (factor GCF x^{-3}first)

C. Rationalize denominator/numerator
3. a. $\dfrac{3-x}{1-\sqrt{x-2}}$

b. $\dfrac{\sqrt{x+1}+1}{x}$

D. Simplify Rational Expressions
4. $\dfrac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$

E. Solve Algebraic Equations and Inequalities
5. Solve.
a. $xy' + y = 1 + y'$ for y'

b. $4x^2 - 21x - 18 = 0$

c. $x^4 - 9x^2 = -8$

d. $\dfrac{7x^2+5x}{x^2+1} - \dfrac{5x}{x^2-6} = 0$

6. Use synthetic division to help factor the following, state all factors and roots/zeros.
a. $p(x) = x^3 + 4x^2 + x - 6$
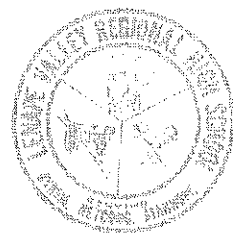
b. $p(x) = 6x^3 - 17x^2 - 16x + 7$

7. Explain (mathematical writing)
a. Explain why $\frac{3}{2}$ cannot be a root of $f(x) = 4x^5 + cx^3 - dx + 5$, where c and d are integers.
   (Hint: Look at the possible rational roots)
b. Explain why $f(x) = x^4 + 7x^2 + x - 5$ must have a root in the interval $[0,1]$.
   (Check the graph and use the sign of f(0) and f(1) to justify your answer.)

8. Solve using a number line/sign chart. Show all work. Give answer sets in interval notation. You may double check with your graphing calculator.

a. $(x+3)^2 > 4$

b. $\frac{x+5}{x-3} \leq 0$

c. $3x^3 - 14x^2 - 5x \leq 0$ (factor first)

d. $x < \frac{1}{x}$

e. $\frac{x^2-9}{x+1} \geq 0$

f. $\frac{1}{x-1} + \frac{4}{x-6} > 0$

g. $x^2 < 4$

h. $|2x+1| < \frac{1}{4}$

9. Solve the system algebraically and then check the solution using your graphing calculator by finding the points of intersection.

a. $\begin{cases} x - y + 1 = 0 \\ y - x^2 = -5 \end{cases}$

b. $\begin{cases} x^2 - 4x + 3 = y \\ -x^2 + 6x - 9 = y \end{cases}$

II. Graphing and Functions
A. Linear Graphs:
10. Write the equation of the line described below.

a. Through (2, 4) and parallel to 2x+3y-8=0

b. Through (1, 2) and perpendicular to 2x+3y-8=0

c. Slope of 5 that passes through (-1 3).
  Also find where this line intersects the x-axis.

d. Passing through (1, -3) and (-2,4).
  Also find where this line intersects the y-axis.

B. Conic Sections:
11. Write the equation in standard form and identify the conic.

a. $x = 4y^2 + 8y - 3$

b. $4x^2 - 16x + 3y^2 + 24y + 52 = 0$

C. Functions:
12. Find the domain and range of the following.

Note: Domain restrictions: denominator $\neq 0$, argument of a log or ln>0, radicand of even index must be ≥ 0
  Domain must be found algebraically.
  Range restrictions: use reasoning, if all else fails, graph it.

a. $y = \frac{3}{x-2}$

b. $y = \log(x - 3)$

c. $y = x^4 + x^2 + 2$
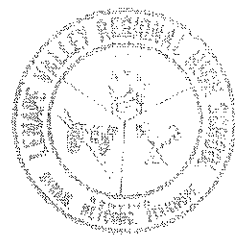
d. $y = \sqrt{2x - 3}$

e. $y = |x - 5|$

f. $y = \frac{\sqrt{x+1}}{x^2-1}$

g. Given f(x) below, graph over the domain [-3,3], what is the range?

$$f(x) = \begin{cases} x & if \ x \geq 0 \\ 1 & if -1 \leq x < 0 \\ x - 2 & if \ x < -1 \end{cases}$$

h. Transformations of functions:

If $f(x) = x^2 - 1$ describe in correct transformation language what the following would do to the graph of f(x).

i. $f(x) - 4$

ii. $f(x - 4)$

iii. $-f(x + 2)$

iv. $5f(x) + 3$
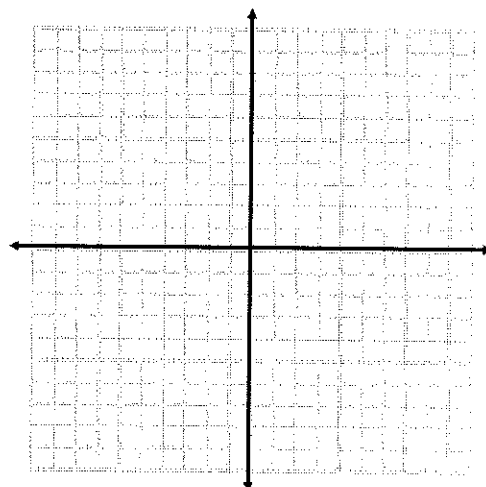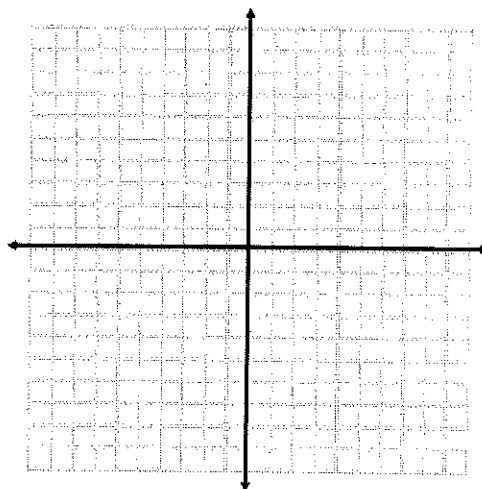
v. $f(2x)$

vi. $|f(x)|$

I. Piece-Wise Functions:

Graph Each. Clearly label points.

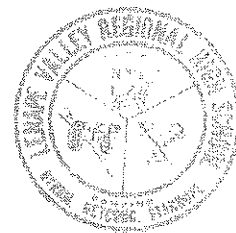i. $f(x) = \begin{cases} 2x & (-\infty, -1) \\ 2x^2 & [-1, 2] \\ -x + 3 & (2, \infty) \end{cases}$

ii. $f(x) = \begin{cases} \sqrt{-x} & -4 \leq x \leq 0 \\ \sqrt{x} & 0 < x \leq 2 \end{cases}$

13. Find the function value, composition, inverses as indicated below. Simplify.

a. Given $f(x) = x^2 - 3x + 4$ **Find** $f(x + 2) - f(2)$.

b. Given $f(x) = |x - 3| - 5$ **Find** $f(1) - f(5)$

c. Given $f(x) = x^2 - 2x - 3$ **Find** $f(x + \Delta x)$

d. Given $f(x) = 8x^2 + 1$ **Find** $\frac{f(x + \Delta x) - f(x)}{\Delta x}$

e. Given $f(x) = \frac{1}{x}$ **Find** $\frac{f(x + h) - f(x)}{h}$

f. Given $f(x) = x - 3$ *and* $g(x) = \sqrt{x}$ **Find:**

    a. $f(g(x))$

    b. $g(f(x))$

    c. $f(f(x))$

    d. $f^{-1}(x)$

    e. $g^{-1}(x)$

    f. $f^{-1}(g(x))$

g. Given $f(x) = \frac{1}{x-5}$ *and* $g(x) = x^2 - 5$ **Find:**

    i. $(f \circ g)(7)$

    ii. $(g \circ f \circ g)(-3)$

    iii. $(g^{-1} \circ g)(x)$ **State the domain**

D. Basic Shapes of Curves:

14. Sketch the graphs. You need to be able to graph the following by knowledge of the shape of the curve, by plotting a few points, by your knowledge of transformations, and without a calculator. You may verify the graph with the calculator to make sure you are correct.

a. $y = \sqrt{x}$ **b.** $y = \ln x$ **c.** $y = \frac{1}{x}$ **d.** $y = |x - 2|$

d. $y = \frac{1}{x-2}$ **e.** $y = \frac{x}{x^2-4}$ **f.** $y = 2^{-x}$ **g.** $y = 3sin\left[2\left(x - \frac{\pi}{2}\right)\right]$

h. $f(x) = \begin{cases} \sqrt{25 - x^2} & if \ x < 0 \\ \frac{x^2 - 25}{x - 5} & if \ x \geq 0, x \neq 5 \\ 0 & if \ x = 5 \end{cases}$

E. Even, Odd, Tests for Symmetry

15. Identify as odd, even, or neither and justify your answer. To justify you must show substitution using –x !!!!! It's not enough to simply check a number.

EVEN: $f(x) = f(-x)$ ODD: $f(-x) = -f(x)$

a. $f(x) = x^3 + 3x$ **b.** $f(x) = x^4 - 6x^2 + 3$ **c.** $f(x) = \frac{x^3 - x}{x^2}$

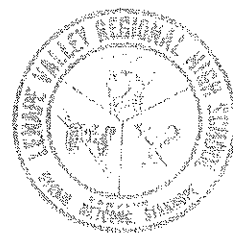**d.** $f(x) = \sin(2x)$ **e.** $f(x) = x^2 + x$ **f.** $f(x) = x(x^2 - 1)$

g. $f(x) = \frac{1 + |x|}{x^2}$ **h.** $y = \cos(x)$ **i.** $y = \frac{|x|}{x+1}$

IV. Logarithmic and Exponential Functions

16. Simplify Expressions without a calculator.

a. $\log_4\left(\frac{1}{16}\right)$

b. $3\log_3 3 - \frac{3}{4}\log_3 81 + \frac{1}{3}\log_3\left(\frac{1}{27}\right)$

c. $\log_9 27$

d. $\log_w w^{45}$

e. $\ln e$

f. $\ln 1$

g. $\ln e^2$

h. $\log_{125}\left(\frac{1}{5}\right)$

17. Solve Equations:

a. $\ln e^x = 4$

b. $\ln x + \ln x = 0$

c. $\ln 1 - \ln e = x$

d. $\ln 6 + \ln x - \ln 2 = 3$

e. $\ln(x + 5) = \ln(x - 1) - \ln(x + 1)$

f. $\frac{e^x + e^{-x}}{2} = 4$

g. $3^{x+1} = 15$

h. $\frac{500}{1+25e^{0.3x}} = 200$

V. Trigonometry

18. Know the unit circle (radian and degree). Be prepared for a test.
Evaluate. Answer must be exact and/or in radians. (You need to be able to do these without a calculator!)

a $\cos(0)$

b. $\sin(0)$

c. $\tan\left(\frac{\pi}{2}\right)$

d. $\cos\left(\frac{\pi}{4}\right)$

e. $\sin\left(\frac{\pi}{2}\right)$

f. $\sin(\pi)$

g. $\arcsin\left(\frac{\sqrt{3}}{2}\right)$

h. $\arctan(1)$

19. State the domain, range, and period of each function.

a. $y = \sin x$

b. $y = \cos x$

c. $y = \tan x$

20. Simplify using Identities.

a. $\frac{\left(\tan^2(x)\right)\left(\csc^2(x)\right)-1}{(\csc x)(\tan^2(x))(\sin x)}$

b. $1 - \cos^2 x$

c. $\sec^2 x - \tan^2 x$

d. $(1 - \sin^2 x)(1 + \tan^2 x)$

21. Solve the Equations over the interval $0 \leq \theta \leq 2\pi$.

a. $\sin^2(\theta) = 1 - \sin(\theta)$

b. $2\tan\theta - \sec^2\theta = 0$

c. $\sin 2\theta + \sin\theta = 0$

d. $2\sin 2\theta = \sqrt{3}$

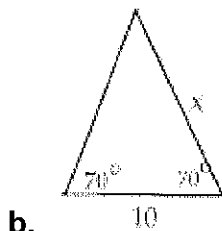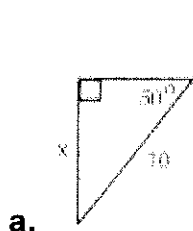**22.** Inverse Trig Functions: Note: $\sin^{-1} x = Arcsin(x)$

a. $\arcsin(1)$

b. $\sin\left(\arcsin\left(\frac{\sqrt{3}}{2}\right)\right)$

c. State the domain and range for: $Arcsin(x)$, $Arccos(x)$, $Arctan(x)$

23. Right Triangle Trig. Find the value of x. (Note: Radian and Degree measure!)

a.   b.   c. use for problem below
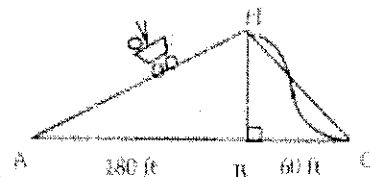
c. The roller coaster shown in the diagram above takes 23.5 sec. to go up the 23 degree incline segment AH and only 2.8 seconds to go down the drop from H to C. The car covers the horizontal distances of 180 feet on the incline and 60 feet on the drop. Decimals in answer may vary.

    i.   How high is the roller coaster above point B?
    ii.   Find the distances AH and HC.
    iii.   How fast (in ft/sec) does the car go up the incline?
    iv.   What is the approximate average speed of the car as it goes down the drop?
    v.   Assume the car travels along HC. Is your approximate answer too big or too small?

24. Graphs: Identify the amplitude, period, Horizontal and Vertical Shifts of these functions.
a. $y = -2\sin(2x)$ **b.** $y = -\pi\cos(\frac{\pi}{2}x + \pi)$

VI. Functions and Models

    25. Be able to do the following on your graphing calculator:

Be familiar with the CALC commands: value, zero, minimum, maximum, intersect.
You may need to zoom in on areas of your graph to find the information.
Answers must be accurate to 3 decimal places. Sketch the graph.

i. Given the function: $f(x) = 2x^4 - 11x^3 - x^2 + 30x$
    a. find all roots (zeros)
    b. find all local maximums
    c. find all local minimums
    d. find f(-1), f(-2), f(0), f(0.125)

ii. Use your calculator to find the solution to the following system: $\begin{cases} y = x^3 + 5x^2 - 7x + 2 \\ y = 0.2x^2 + 10 \end{cases}$

26.   Problem Solving:

a. Find the surface area of a box of height h whose base dimensions are p and q, and that satisfies the following condition.

   i. The box is closed.

   ii. The box has an open top.

   iii. The box has an open top and a square base with side length p.

b. A seven foot ladder, leaning against a wall, touches the wall x feet above the ground. Write an expression (in terms of x) for the distance from the foot of the ladder to the base of the wall.

c. A piece of wire 5 inches long is to be cut into two pieces. One piece is x inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the square and the circle as a function of x.

d. Biologists have noticed that the chirping rate of crickets of a certain species is related to temperature, and the relationship appears to be very linear. A cricket produces 113 chirps per minute at 70°F and 173 chirps per minute at 80°F.

   i. Find a linear equation that models the temperature T as a function of the number of chirps per minute N.
   ii. What is the slope of the graph? What does it represent?
   iii. If the crickets are chirping at 150 chirps per minute, estimate the temperature.

e. Under ideal conditions a certain bacteria population is known to double every three hours. Suppose that there are initially 100 bacteria.

   i. What is the size of the population after 15 hours?
   ii. What is the size of the population after t hours?
   iii. Estimate the size of the population after 20 hours.
   iv. When will the population reach 1250 bacteria?

VII. Limits

27. Evaluate each. Show work or explain your answer:

   i. $\lim_{x \to 3} x^2 + 2$

   ii. $\lim_{x \to -3} \frac{(x+3)(x-4)}{(x+3)(x+1)}$

   iii. $\lim_{x \to 25} \frac{\sqrt{x}-5}{x-25}$

   iv. $\lim_{x \to -2} \frac{x-4}{x^2-2x-8}$

   v. $\lim_{x \to -3} \frac{x^2+2x-3}{x^2+7x+12}$

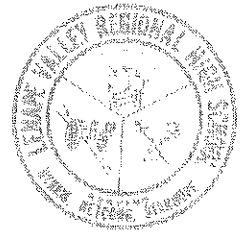   vi. $\lim_{x \to -2} \frac{x^3+8}{x+2}$

   vii. $\lim_{x \to 5} \frac{x-5}{|x-5|}$

   viii. $\lim_{x \to 8} \frac{1}{x-8}$

   ix. $\lim_{x \to -\infty} \frac{x-4}{x^2-2x-8}$
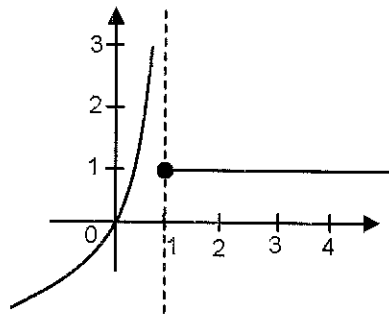
   x. $\lim_{x \to \infty} \frac{x^2+2x-3}{x^2+7x+12}$

28. One-sided limits:

For each of the following
determine:

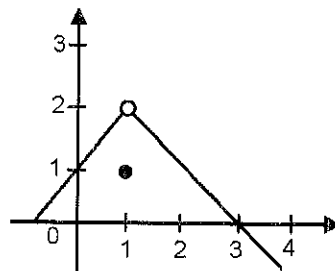**a**. $\lim_{x \to 1^-} f(x)$ **b**. $\lim_{x \to 1^+} f(x)$ **c**. $\lim_{x \to 1} f(x)$

i.

ii. $f(x) = \begin{cases} x^2 - 1 & x < 1 \\ 4 - x & x \geq 1 \end{cases}$

iii.

iv. $f(x) = \begin{cases} -x^2 & x < 1 \\ 2 & x = 1 \\ x - 2 & x > 1 \end{cases}$